PRICING ON TWO-SIDED MATCHING PLATFORMS WITH HORIZONTALLY DIFFERENTIATED AGENTS

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Abstract. This paper is the first in a series of papers that explore pricing on two-sided matching markets with horizontally-differentiated agents in the context of online dating platforms. In this paper, a monopolist provider serves two sides of the market (men and women) whose characteristics are uniformly distributed on the unit circle. The paper explores both uniform pricing and two-part tariffs. The two main results are that there exists a symmetric equilibrium where agents find matches in finite time and that optimal pricing involves charging a uniform price when the agents first enter the platform and charge zero per-interaction prices afterwards.

1. Introduction

The study of two-sided markets is one that is relatively new in the area of industrial organization. A two-sided market is a market where agents are brought together by a platform or network. We generally assume that these agents are segmented into two “sides”. Agents on each side usually exhibit increasing utility in the number of agents on the opposite side (positive inter-group network externalities) and may exhibit decreasing utility in the number of agents on the opposite side (negative intra-group network externalities). Platforms function as an intermediary and make profit charging a fee to one side in order to provide access to agents on the other side. The prices set by the platforms may be different for both sides. For example, a nightclub that is trying to attract women may charge a price of zero (free access) to the women but charge a positive price to the men.

There has been a debate in the literature as to why a two-sided market matters. In particular, what can two-sided markets explain that a one-sided market cannot? Rochet and Tirole (2006) argue that a two-sided market should be defined as one in which the volume of transactions between end-users depends on the structure of the market and not only on the overall level of the fees charged by the platform. In particular, a platform’s value is really the benefits or access that it is able to provide to the agents. They stress that “a platform’s usage or variable charges impact the two sides’ willingness to trade once on the platform and, thereby, their net surpluses from potential interactions; the platforms’ membership or fixed charges in turn condition the end-users’ presence on the platform.” The effectiveness of a platform as an intermediary depends heavily, of course, on the fact that the two sides of the market cannot easily get together.

There are many pricing schedules that are offered by two-sided platforms. Armstrong (2006) characterizes three main factors that determine the structure of prices offered to the two groups. The first and most important is the relative size of inter-group externalities. Suppose that the two sides of the market are sides A and B. If side A exerts a large positive inter-group externality on side B, the platform will aggressively target agents on side A. In a competitive market setting, side A’s benefits to side B determines side A’s pricing (not how much side A benefits from the presence of side B). Secondly, a common choice of pricing structures is the choice of whether to employ a fixed fee or a per-transaction charge or both (two-part tariff). In particular, platforms might charge for their services on a lump-sum (fixed) basis, so that an agent’s payment does not explicitly depend on how well the platform performs in attracting agents from the other side. On
the other hand, a platform may want to charge per-interaction fees in the case where the platform has successfully managed to court a good distribution of agents on the opposite side. The final characteristic that may determine the pricing structure on platforms is whether or not agents on both sides single-home or multi-home. Agents who single-home choose to use only one platform while agents who multi-home use many platforms.

There are many examples that come to mind when talking about two-sided markets. One well-researched example is a credit card network; for example, Visa cards. The two sides of a credit card platform or network are the consumers (users) and merchants (sellers). The more merchants accept Visa cards and the more membership benefits offered, the more attractive the network becomes to the users. Alternatively, merchants prefer to join credit card networks that have a large subscription of consumers. Another example of a platform in a two-sided market is a company that facilitates B2B (business-to-business) transactions. The online website Alibaba.com is a good illustration of a two-sided market. Alibaba, a Chinese company, functions as a medium to bring businesses (suppliers, wholesalers, manufacturers) together. It essentially functions as a trading platform for businesses of all sizes. An example of a two-sided market that has not been as widely researched as others is online dating websites. This market will be the focus of this paper. Heterosexual online dating websites function as platforms to bring together women and men intending to find matches. Online dating firms are extremely valuable to agents because the search costs for a mate are assumed to be very high in real life. Thus, online dating platforms can alleviate this inconvenience by bringing people together in order for them to obtain a match.

The online dating industry is extremely successful with total revenues from 2012 reaching $2.1 billion.\(^1\) One of the biggest players in the online dating market, eHarmony, reported that they were responsible for nearly 5% of marriages in the U.S in the year 2009.\(^2\) The industry in general is doing extremely well but a particular subset of dating websites has become extremely popular in recent years. This subset is called niche dating platforms. Niche dating platforms are platforms that cater to agents who are interested in matching up with partners with very specific attributes. For example, the website VeggieDate caters to vegetarians and the website JDate provides access to Jewish singles. These websites operate on a somewhat different concept that general interest dating websites like eHarmony, who caters to a wider range of clients. Niche dating platforms serve two main functions. The first is to further reduce the search costs of finding a match. For example, if a Catholic woman wanted to narrow down her scope in finding a match to only Catholic men, she would be faced with a better distribution of people if she joined a Catholics-only website versus one that has men from all religious backgrounds. The second function is to partition heterogeneous consumers into categories of attributes which they feel the strongest about which forces them to reveal their private information about themselves. An online platform can then presumably charge higher prices to agents who reveal that they have high willingness-to-pay for good matches with agents who characteristics are closely related to their own.

One question that then arises is one of efficiency. Is it more or less efficient to have agents partitioned into small platforms according to their characteristics as compared to having agents all join one large network and self-select into matches? This paper takes the first step to address this question. A second issue that we would like to address relates to optimal pricing. There are vast differences amongst the pricing structures for online dating websites. Online dating websites usually provide a free trial run to agents on both sides but charge a membership fee after a certain amount of time or after a certain amount of interactions with agents from the other side of the market. Most platforms charge a fixed fee for a specific term (1 month, 3 months, 6 months) but some platforms charge per-interaction fees or use a credit system. For example, eHarmony charges

\[^2\text{www.eharmony.com}\]
per-month prices that range from $19.95 to $59.95 per month. This paper also addresses the issue in the context of the model provided. It turns out that the optimal pricing in our context will be to charge a uniform fixed price when the agents first enter the platform and to charge zero per-interaction fees afterwards.

The organization of the paper is as follows. Section 2 will be devoted to a literature review on the subject. In Section 3, we will introduce our model in the context of uniform pricing and derive some results. Section 4 further extends the uniform pricing to the case where the monopolist charges two-part tariffs, Section 5 provides the conclusion to the paper and an introduction to future work and Section 6 is the appendix.

2. Literature Review

The definition of the two-sided literature itself can be very broad. Rysman (2009) argues that the literature is “distinguished by its focus on the actions of the market intermediary.” Research in two-sided markets generally explores decisions of platforms who act as market intermediaries, in the presence of some sort of interdependence or externalities between the two sides that the platform serves. This externality could of course involve usage or membership to the platform. For example, Rysman (2009) posits that merchants on a credit card platform do not care about how many other merchants on the platform there are but the merchants do care about how many customers are subscribing to the platform. In a sense, a platform’s attractiveness and value is derived completely from its ability to provide each side with interactions with agents from the opposite side. This is a common theme in the two-sided literature: most work is done on inter-group externalities and do not focus on much on competition within a side to obtain interactions with agents on the other side. In fact, it seems as though this subset of the literature may lead to a very fertile ground of research, since many two-sided markets possess this characteristic of competition within a side. For example, it’s very easy to see a dating site exhibits this behavior: even though there may be many agents on the opposite side, agents on any particular side face competition from agents on their side to obtain a good match.

One of the seminal papers in the two-sided market literature is Armstrong (2006). Armstrong places a strong emphasis on the effects of inter-group network externalities on pricing. In particular, as stated in the introduction, he posits that the three main factors of pricing structures from the point of view of a network are the relative size of inter-group externalities, the choice of whether to employ a fixed fee or a per-transaction charge, and whether or not agents on each side single-home or multi-home. He expands on earlier work by Rochet and Tirole (2003) by modifying the specifications of agents’ utility, the structure of platforms fees, and the structure of platforms’ costs. Armstrong presents three models of two-sided markets: monopoly platforms, competing platforms where agents join a single platform and ‘competitive bottlenecks’ where one group joins all platforms.

Weyl (2010) makes great progress in further studying heterogeneity in two-sided platforms by developing a theory of monopolistic platform pricing. In his paper, Weyl extends the basic framework of Rochet and Tirole (2003) and proposes a more plausible (yet equally tractable) model of heterogeneity in which agents differ in their income. Weyl’s model incorporates a continuum of users and gives a general measure of market power in order to study policy questions. These policy questions include those of price regulation and dealing with mergers of platforms. Weyl also studies socially optimal pricing for a monopolistic platform and compares it to Pigouvian, profit-maximizing and Ramsey prices.

There are of course many variations to the main frameworks employed in the two-sided market literature. For example, Hagiu (2006) studies two-sided platforms in which the sellers and buyers on the platform do not arrive at the same time. For example, in the development of software and

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3For a 12−month plan, the cost is $19.95 per month. For a 6−month plan, the cost os $29.95 per month. For a 3−month plan, the cost is $39.95 per month. For a one-month plan, the cost is $59.95 per month.
video games, application and game sellers usually join a platform before most buyers do. Hagiu states that this is more than often due to technological reasons: the development of video games and applications is often a lengthy and costly process, so platforms take a lot of time to prepare their products before starting to market them to the public. This then causes the network effects to be asymmetric, precisely because buyers will simply subscribe to the platform which provides them with the highest utility or surplus, given the prices of all other platforms and the platform itself. The coordination and Bertrand game is then played only by the sellers. Hagiu then goes on to studying both monopolistic and competitive platform structures. Cabral (2011) considers a dynamic model of competition between networks, where consumers die and firms continuous compete Bertrand-style for new consumers. He then goes on to studying termination charges within wireless communications networks, where customers and eligible for lower prices for a set period and then pay higher prices once that period is over.

The most seminal paper in the matching literature is undoubtedly the paper by Gale and Shapley (1962). In a two-sided matching model, agents often have a preference over all the agents on the other side from whom they would form a match with. Hoppe, Moldovanu and Sela (2008) work off the basic assortative matching framework and present a model of costly signaling with heterogeneous agents. They argue that the act of signaling comes with a large benefit where agents are able to maximize total surplus generated from matching but this benefit is reduced by the costs of signaling. The main purpose of this paper then is to study this tradeoff in the context of a two-sided matching market. Hoppe, Moldovanu, and Sela first consider a bidding model with finite number of agents on both sides of the market where agents on one side represent the prizes for which the agents on the other side compete. They then study this limit model and effects of policies that attempt to curb “wasteful signaling” (i.e., costly signaling that serves no purpose and does not contribute to increasing efficiency).

Kojima and Pathak (2009) study stability in two-sided matching markets. A matching is defined as stable if there is no individual agent who prefers to become unmatched or pair of agents who prefer to be assigned to each other to being assigned their allocation in the matching. They state that empirical research has shown that stable mechanisms often succeed, whereas unstable ones often fail. Kojima and Pathaks’ findings are that under some regularity conditions, the proportion of agents with incentives to lie about their preferences when all other agents are truthful approaches zero as the market becomes very large. Azevedo and Leshno (2012) found that in both the discrete and continuum two-sided matching markets with heterogeneous agents, stable matchings have a very simple structure, with matches being formed if agents on each side are ranked above a certain threshold.

The work done in this paper most closely relates to work done by Halaburda and Piskorski (2011), and Damiano and Li (2007). In a recent working paper, Halaburda and Piskorski (2011) explored competition amongst search platforms. In particular, she showed that a two-sided platform can successfully compete by limiting the choice of potential matches it offers to its customers while charging a higher price than platforms with unrestricted choice. Halaburda and Piskorski posit that there has been, up till now, a gap between the theoretical and empirical literature regarding search platforms. In particular, theory has suggested that the most efficient matching platform structure is one that offers unrestricted access to its participants. In reality, however, we observe that there are many restricted platforms that coexist with unrestricted platforms; in fact, these restricted platforms are prospering and have the ability of charging higher prices. Halaburda and Piskorski capture this in their model by assuming that agents have heterogeneous outside options. This is different than the normal assumption that agents have an outside option of 0 if they do not form a match. Halaburda and Piskorski find that increasing the number of agents on each side of the market not only has a positive effect due to large choice (i.e. positive inter-group network externalities), but also a negative effect due to competition between agents on the same side (i.e. negative intra-group network externalities). Agents resolve the tradeoff between these
externalities differently; in particular, agents with low outside options face stronger competitive effects that they do choice effects. Thus, these agents have a higher willingness to pay for a platform restricting choice, and vice versa.

Damiano and Li (2007) study the problem of a monopoly matchmaker that uses a schedule of entrance fees to sort different types of agents on the two sides of a matching market into exclusive ‘meeting places’, where agents randomly form pairwise matches (and obtain a match with probability 1). These meeting places, in the case of online dating websites, can be interpreted as different online dating databases. They then go on to providing necessary and sufficient conditions for the revenue-maximizing sorting to be efficient. In particular, these conditions require that the match value function exhibits complementarities in types. The complementarity condition is embedded in the multiplicative match value function; that is, assuming that a female is of type $x$ and a male is of type $y$, a match value function of $xy$ means that each agent’s willingness to pay for an improvement in match type increases with the type of the agent.

The model presented in this paper combines elements from the matching and pricing literatures. In particular, it extends the Salop model to incorporate matching. The Salop (1979) model is a variant of the Hotelling (1929) model of spatial competition. Salop introduces an economy consisting of two industries: a monopolistically competitive industry with differentiated brands and decreasing average costs and a competitive industry producing a homogeneous commodity. There are precisely $L$ number of consumers who either buy a unit or none of the differentiated brand according to preferences, prices and the distribution of brands in the product space. Each agent’s remaining income is spent on the homogenous commodity. Each agent is assumed to have their most-preferred brand specification. Any brand deviation from the most-preferred brand results in a lower utility level. The product space is assumed to either be an infinite line or a unit-circumference circle. Salop states that “neither assumption is realistic, but avoids the difficulties of corner solutions of the Hotelling model.” This enables him to analyze the intuition of the model without algebraic complications. Agents incur a transportation cost for buying a brand that deviates from his or her most-preferred brand and this then influences the pricing structure. This model provides a benchmark for subsequent analyses with non-uniform preferences.

### 3. Uniform Pricing

In this infinite horizon, discrete-time economy there is a single monopolist who operates one platform. The two sides to the platform are men and women. Agents have characteristics which are uniformly distributed on the circumference of the unit circle $[0, 1]$. The distribution of characteristics is common knowledge. Searching for a match is assumed to be costly. The monopolist platform functions to bring men and women together to reduce this cost but also charge a fixed price $p_t$ in time $t$ for this convenience.

**Timing, Inflows and Outflows.** We can think of each side of the platform as having two “groups” that consist of those who are already on the platform and those who are deciding whether or not to join the platform. On each side and in each time period $t$, we assume that there is a mass of size 1 of agents who decide whether or not to join the platform. Agents enter the platform at rate $e_t$. New agents are then born outside of the platform at rate $b_t$ to replace those who have left. At the start of each period, there is a mass $x_t$ of agents who are already on the platform. When agents find matches, they leave the platform at rate $\mu_t$. Reiterating, we have that the timing of the model is as follows. In one time period $t$, the following events happen in order:

1. Agents who are outside of the platform decide whether or not to join the platform. They join at rate $e_t$.
2. Agents who find matches leave the platform at rate $\mu_t$.

Each agent who is left is replaced with their identical clone. This is crucial to ensure that the distribution of agents remains the same throughout time.
(3) Agents are born at exogenous birth rate $b_t$ outside of the platform to replace those agents who have left.

The transitional equation that describes the mass of people on the platform is

$$x_{t+1} = (1 - \mu_t)x_t + e_t$$

Note that the total rate of inflow into the platform is $e_t$ and the total rate of outflow out of the platform is $\mu_t$.

**Agent’s Problem.** In each period, agents who are not on the platform face the decision of whether or not to enter the platform. Faced with the fixed fee $p_t$ that is charged by the monopolist, agents choose to join if and only if their expected utility of being on the platform outweighs their reservation utility (which is normalized to 0). If agents choose to join the platform by paying the price $p$, they got through the following motions (in order) within one period:

1. Agents obtain a random draw of a new date.
2. Agents learn each others’ characteristics during the date.
3. Agents decide whether or not to form a match. If they both decide to form a match, they each obtain their matching utility and leave the platform forever. If one or both of them decide not to match, they continue searching by going into the next period.

The utility\(^5\) obtained by agent $x$ when he chooses to match with agent $y$ is

$$u^x(y) = b - c \mid x - y \mid$$

where $b > 0$ is the benefit the agent obtains from matching and the distance measure $\mid$ denotes the shortest arc-length between $x$ and $y$. All agents discount time at a rate $\delta \in (0, 1)$. The cost of a bad match is represented by $c > 0$.

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\(^5\)This form of the utility function is commonly used in the two-sided platform literature.
Definition 1. (The agent’s Bellman equation) The Bellman equation for agent $x$ is given as

$$V_x(\cdot) = \max\{u^x(\cdot), (1 - \delta) \int_0^1 V_x(\cdot) d(\cdot)\}$$

where $\delta \int_0^1 V_x(\cdot) d(\cdot)$ is the expected continuation utility.

Agents have an “ideal type” that they would like to matched to. For simplicity, we define the ideal type for type $x$ to be type $x$. In solving their optimization problem, agents will use a threshold strategy in equilibrium. This means that agents will choose an $\epsilon$ so that an agent of type $x$ will form a match with types within the interval $[x - \epsilon, x + \epsilon]$ and keep on searching if his or her random draw is within the interval $[x - \epsilon, x + \epsilon]$. Using the Bellman equation given above, we may solve for the $\epsilon$-threshold of each agent.

Proposition 2. The $\epsilon$-threshold for each agent is given by

$$\epsilon = \min\left\{ \frac{1}{2}, c(\delta - 1) + \sqrt{c(\delta - 1)(c(\delta - 1) - 4b\delta)} \right\}$$

Proof. Appendix. □

There are a couple of simple implications from Proposition 2. First of all, note that the $\epsilon$-threshold for agents is independent of types. Also, regardless of the strategy of the opposing player $j \neq i$, $i$’s best response is always to use an $\epsilon$-strategy. This fact is proven bellow in a following lemma. Since each player’s $\epsilon$ is the same, symmetry implies that if an agent $x$ wishes to form a match with agent $y$, then agent $y$ will want to form a match with agent $x$. In particular, if $S_x$ is defined to be the set of possible matches for agent of type $x$, then $S_x = \epsilon$.

The only reason why we have attained this is result is the assumption of uniformity in types. Without the uniform distribution of types, it is not always going to be true that matching happens in such a trivial manner. This question of what will happen in the case when $\epsilon$ is type-dependent will be explored in the next paper.

Proposition 3. Given any other strategy employed by $y \neq x$, $x$’s best response will always be an $\epsilon$-threshold.

Proof. The strategy for player $x$ is determined by his or her optimization problem in Equation (3.3). The expected continuation utility of $x$ does not depend on the type of agent $y$ but his discounted matching utility $u^x(\cdot)$ strictly decreases in the distance between $x$ and $y$. This being, there must be a threshold type $\bar{y}$ for which $x$ is indifferent between dating and not. This type $\bar{y}$ will define a threshold strategy for $x$. □

For ease of notation, define $\bar{\epsilon} = \frac{c(\delta - 1) + \sqrt{c(\delta - 1)(c(\delta - 1) - 4b\delta)}}{\delta}$. For the case when $\bar{\epsilon} < \frac{1}{2}$, we may analyze the effects of the parameters of the model on $\epsilon$.

Proposition 4. In the case where $\bar{\epsilon} < \frac{1}{2}$, we have that $\frac{\partial \epsilon}{\partial b} > 0$, $\frac{\partial \epsilon}{\partial \delta} < 0$, and $\frac{\partial \epsilon}{\partial c} < 0$.

Proof. Appendix. □

Proposition 3 provides three intuitive and results about the $\epsilon$-strategy employed by agents. In particular,

1. The higher the benefits from matching are, the less picky agents become about which agents they choose to match with.
2. The higher the agents value the future, the more picky they become. This makes sense because it means that they do not mind waiting longer for a better match.
3. The higher the disutility is from a bad match, the more picky an agent becomes.
Monopolist’s Problem. The monopolist maximizes profits in each time period $t$. In particular, he faces the following optimization problem.

$$
\max_{\{p_t\}_{t \geq 0}} \sum_{t=0}^{\infty} \delta^t 2 p_t x_t \\
\text{s.t.}\quad (\text{IR}): \quad \delta \int_0^1 V^x(.) d(.) - p_t \geq 0 \quad \forall \text{ types } x \text{ and } \forall t \geq 0
$$

In each time period $t$ there is a mass $x_t$ of agents on each side of the market. Each agent is charged $p_t$; hence the profit that the monopolist obtains in each time period is $2 p_t x_t$. The constraint the monopolist faces is an individual rationality constraint that ensures that agents will join the platform. Since each agent faces the same optimization problem, each type has the same individual rational constraint. The monopolist will set a price $p_t$ to encourage all types to join, implying that the price $p_t$ is constant in each period (since the parameters of the model are time-invariant). In particular,

$$
p_t = p = \delta \int_0^1 V^x(.) d(.) \quad \forall t
$$

By evaluating the integral on the right-hand side of Equation 3.5 and substituting in for values of $\epsilon$ obtained from (3.4), we have that

$$
p = \frac{c(1 - \delta) + 2b\delta - \sqrt{c(\delta - 1)(c(\delta - 1) - 4b\delta)}}{2\delta(1 - \delta)}
$$

Proposition 5. The fixed price charged by the monopolist is always strictly positive.

Proof. Appendix.

Steady-State Conditions. The steady state conditions for this model can be characterized by a couple of equations. First of all, it must be that $e_t = b_t \forall t$ since we assume that in each time period agents who leave to join the platform are replaced by an identical group of agents who are born at the end of the period. Secondly, by dropping subscripts in Equation (3.1) we obtain the following equation

$$
x = (1 - \mu)x + e \\
\iff \mu x = e \\
\iff x = \frac{e}{\mu}
$$

(3.7)

We also have that since the monopolist sets $p$ to extract all the surplus from all agents, it has to be that in steady-state, $e = 1$. Also, given that agent types are uniformly distributed, the rate at which the agents find matches and leave will be exactly $2\epsilon$. Thus, we have that $\mu = 2\epsilon$. Substituting into Equation 3.7 yields

$$
x = \frac{1}{2\epsilon}
$$

(3.8)

which is constant, given that $\epsilon$ is time and price-invariant.

Equilibrium. The notion of equilibrium that we will be using a symmetric equilibrium in steady state. The existence of the equilibrium is proven in the next proposition. In particular, all agents are able to find matches in finite time.

Theorem 6. A symmetric equilibrium exists where all agents find a match in finite time.
Proof. Given the price \( p \) charged by the monopolist, it is clear that everyone (all agents from both sides) will participate. It is only a matter of whether all the agents will find a match. Since all agents are identical, they have the same \( \epsilon \). Since the distribution of agents on both sides is uniform, the probability of finding a match will just be the proportion of the \( \epsilon \)-interval out of the entire length of circle. That is,

\[
P(\text{finding a match}) = \frac{2\epsilon}{1} = 2\epsilon
\]

The setup of the model implies that each period is identical and independent. Thus, the probability that an agent has yet to find a match at time \( t \) is

\[
P(\text{not found a match by time } t) = \prod_{k=1}^{t} P(\text{not finding a match}) = (1 - 2\epsilon)^t
\]

Noting that \( 2\epsilon \leq 1 \), we have that \( (1 - 2\epsilon) \leq 1 \). Thus as \( t \to \infty \), the probability that an agent has not found a match goes to 0. Thus, agents will be able to find matches in a finite time. □

4. Two-Part Tariffs

Some online dating platforms like JustForLunch employs a two-part tariff pricing structure in which agents are charged a specific fixed price for access to the services provided by the platform and additional “per-date” charges for each date the agents go on. In this section of the paper, we will extend the baseline model to allow the monopolist platform to not only charge a fixed price \( p \) in each period but also a per-interaction price \( \bar{p} \).

The basic framework is exactly the same as in Section 3. In particular, the timing of the inflows and outflows on the platform are the same. However, each agent’s continuation utility on being on the platform is now dependent on the price \( \bar{p} \) that must be paid in order to receive a new draw and continue to the next round.

Definition 7. (The agent’s Bellman equation) The Bellman equation for agent \( x \) is given as

\[
V_x(.) = \max \left\{ \frac{u^x(.)}{1 - \delta}, \delta \left( \int_0^1 V_x(.) \ d(.) - \bar{p} \right) \right\}
\]

where \( \delta \left( \int_0^1 V_x(.) \ d(.) - \bar{p} \right) \) is the expected continuation utility.

As done in the previous section, we may find a corresponding \( \epsilon \) for each agent. This \( \epsilon \) is given in the next proposition.

Proposition 8. The \( \epsilon \)-threshold for each agent is given by

\[
\epsilon = \min \left\{ \frac{1}{2}, \frac{c(\delta - 1) + \sqrt{c(\delta - 1)(c(\delta - 1) - 4\delta(b + \bar{p}\delta))}}{2c\delta} \right\}
\]

Proof. Appendix. □

For simplicity of notation, let \( \bar{\epsilon} = \frac{c(\delta - 1) + \sqrt{c(\delta - 1)(c(\delta - 1) - 4\delta(b + \bar{p}\delta))}}{2c\delta} \). Note that \( \bar{\epsilon} \) is now dependent on \( \bar{p} \). In particular, to ensure that \( \epsilon \in \mathbb{R}_{++} \), we have to impose an assumption on the possible values of \( \bar{p} \).

Remark 9. We assume from here onwards that the optimal \( \bar{p} \geq -\frac{b}{\delta} \).

Note that, given this specification, we are allowing the optimal \( \bar{p} \) to be negative. Intuitively, a negative per-interaction price would imply a subsidy provided by the platform. For example, a nightclub could charge a fixed fee to enter the club and declare some days as “ladies night” when drinks are free for all women.

Since we have an explicit form for \( \epsilon \), as in the previous section, we can do some simple comparative statics. The behavior of \( \epsilon \) with respect to changes in \( \delta, b \) and \( c \) are the same as in the previous section. The only different result that we will obtain is one that involves \( \bar{p} \).
Proposition 10. In the case where $\bar{\epsilon} < \frac{1}{2}$, we have that $\frac{\partial \bar{\epsilon}}{\partial \bar{p}} > 0$.

Proof. Appendix. $\square$

The reasoning behind why a higher per-interaction price increases $\bar{\epsilon}$ is simple. A larger $\bar{\epsilon}$ corresponds to the agent being less picky about the types that he matches with. A higher $\bar{p}$ increases the costs of staying in the platform (and so lowers the continuation utility of being on the platform) since agents have to pay $\bar{p}$ in order to receive a new draw for a date. This makes agents internalize the extra costs by expanding the set of acceptable matches by choosing a large $\bar{\epsilon}$ in order to leave the platform quicker.

The monopolist’s problem now is slightly different as he is able to charge a fixed-price only to the new cohort who joins. From the setup of the model, we have that an agent who joins the platform in time $t$ is able to also obtain a draw of a new date in the same period. Thus the optimization problem solved by the monopolist in each period is

$$\max_{(p_t, \bar{p}_t)} \sum_{t=0}^{\infty} \delta^t 2(\bar{p}_t x_t + (p_t + \bar{p}_t)\epsilon_t)$$

s.t. IR$_x^t$ holds for all types $x$ and all time periods $t$.

The objective function of the monopolist is derived from the fact that the monopolist now serves two cohorts in any period $t$: the old cohort that is already on the platform and the new cohort who just entered. The monopolist is able to charge both the fixed price and the per-interaction price to the new cohort but can only charge the fixed price to the old cohort. We need to find an explicit form for Equation (4.1). To do this, note that there are two states for an agent of type $x$: he either finds a match in this period (in which case he leaves the platform and matches forever) or he does not and must continue searching (and pays $\bar{p}$ to go to the next period). Given the uniform distribution of types on $[0, 1]$, we have that the probability of an agent obtaining a match in any period is merely the length of his or her $\epsilon$-interval, which is $2\epsilon$. Thus the expected discounted utility $U^x$ of being on the platform is recursively determined as follows.

$$U^x = 2\epsilon \int_{x-\epsilon}^{x+\epsilon} \frac{u^x(y)}{1-\delta} \ dy + (1-2\epsilon)\delta U^x - \bar{p}$$

where $u^x(y) = b - c \mid x - y \mid$. Solving for $U^x$ yields

$$U^x = \frac{2\epsilon}{(1-\delta)(1-\delta(1-2\epsilon))} \int_{x-\epsilon}^{x+\epsilon} u^x(y) \ dy - \frac{\bar{p}_t}{(1-\delta(1-2\epsilon))}$$

Since $\int_{x-\epsilon}^{x+\epsilon} u^x(y) \ dy = 2b\epsilon - c\epsilon^2$ and $\epsilon$ is type independent, we have that $U^x = U$ for all $x$. Consequently, the individual rationality constraint faced by the monopolist in each period is

$$\text{(IR)}^t U - p_t \geq 0$$

In each period $t$, we have that the fixed price $p_t$ will be set to extract all surplus from agents. Our analysis is complicated by the fact that $\epsilon$ now depends on $\bar{p}_t$. In order to understand the relationship between $\bar{p}_t$ and $p_t$, we must understand how $U$ changes with respect to changes in $\bar{p}_t$. Intuitively, as stated before, the expected discounted utility of being on the platform should decrease with the introduction of a positive $\bar{p}_t$. The attractiveness of the platform as a means of obtaining a match decreases as it is now more costly to remain on the platform for longer. This result is stated in the following proposition.

Proposition 11. The expected discounted utility of being on the platform, $U$, decreases as $\bar{p}_t$ increases.
Proof. We know that as $\bar{p}_t$ increases, $\epsilon$ increases as well (Proposition 10). In fact, as $\bar{p}_t$ increases, $\epsilon$ approaches $\frac{1}{2}$. In particular, as $\epsilon \to \frac{1}{2}$, we have that

$$U(\bar{p}_t) \to \frac{b - \epsilon - \bar{p}_t}{(1 - \delta)}$$

which is strictly decreasing in $\bar{p}_t$. □

As an easy consequence of the fact that in each time period the fixed price $p_t$ is set to that $U = p$, we have the following corollary.

**Corollary 12.** The fixed price $p$ is inversely related to the per-interaction price $\bar{p}$.

**Steady-State Equilibrium.** In a steady-state equilibrium, we have that Equation (3.8) still holds. In particular, by Proposition 11, we have the following result.

**Theorem 13.** In a steady-state equilibrium, the monopolist charges a zero per-interaction price $\bar{p}$ and sets the fixed price $p = U$, where $U$ is given in Equation (4.2).

This theorem directly follows from the fact that $U$ strictly decreases in $\bar{p}$. The introduction of a positive per-interaction price causes agents to choose a suboptimal $\epsilon$ (relative to the uniform pricing case) in order to internalize the extra costs of being on the platform. Since the ability of the monopolist to extract surplus in the form of the fixed price depends on $U$, his ability to charge a high $p$ decreases as he charges a higher $\bar{p}$. Since the monopolist discounts at the same rate as the agents, he prefers to charge a lump-sum $p$ now instead of a smaller $p$ now and very small streams of $\bar{p}$ later. Thus, we obtain that the equilibrium obtained in the case of two-part tariffs is exactly identical to that of uniform pricing. It should also be pointed out that the existence of such a symmetric steady-state equilibrium follows from Theorem 6.

5. Conclusion

This paper provides a matching model which incorporates prices. The two main results from this paper are that a symmetric equilibrium exists where agents find a match in finite time and that the optimal pricing strategy is to charge a positive uniform price when agents first join the platform and then charge zero for any interactions or dates that the agents may have later. The model is simplistic and yields intuitive results.

The next step is to build on this model to incorporate more friction into the interactions to obtain more interesting dynamics. We still have not answered the question of how breaking apart a large platform into smaller niche platforms affects pricing and efficiency. The next paper in this line of research will be feature pricing when a monopolist operates many smaller platforms in the presence of agents who are horizontally differentiated.
6. Appendix

Proof of Proposition 2.

Proof. To find the $\epsilon$-threshold for an agent $x$, we need to find the type $\bar{y}$ for which $x$ is indifferent between forming a match with and not. That is,

$$\frac{u^x(\bar{y})}{(1-\delta)} = \delta \int_0^1 V(y) \, dy \quad (6.1)$$

Assuming that the agent of type $x$ uses an $\epsilon$-threshold strategy, we have that $|\bar{y} - x| = \epsilon$. We also have that the right-hand side of the previous equation may be rewritten as

$$\frac{\delta}{(1-\delta)} \int_{x-\epsilon}^{x+\epsilon} (u^x(y) - u^x(\bar{y})) \, dy + \int_0^1 u^x(\bar{y}) \, dy$$

because agents receive the same utility outside of the $\epsilon$--interval since they choose not to match. Also, note that the symmetry of the $\epsilon$--interval allows for much simplification. Thus, (6.1) may be rewritten as

$$b - c\epsilon = \frac{\delta}{(1-\delta)} (2 \int_{x-\epsilon}^{x+\epsilon} (\epsilon - (y - x)) \, dy + (b - c\epsilon)) \quad (6.2)$$

Solving (6.2) for $\epsilon$ yields

$$\epsilon = \frac{c(\delta - 1) + \sqrt{c(\delta - 1)(c(\delta - 1) - 4b\delta)}}{2c\delta}$$

where $\epsilon > 0$.\(^6\) To avoid the problem of having an $\epsilon$--interval that wraps more than once around the circle, we impose the condition that $\epsilon \leq \frac{1}{2}$, as stated in the proposition.

Proof for $\bar{\epsilon} > 0$ in the uniform pricing case.

Proof.

\[
\bar{\epsilon} = \frac{c(\delta - 1) + \sqrt{c(\delta - 1)(c(\delta - 1) - 4b\delta)}}{2c\delta} > 0 \\
\iff c(\delta - 1) + \sqrt{c(\delta - 1)(c(\delta - 1) - 4b\delta)} > 0 \\
\iff \sqrt{c(\delta - 1)(c(\delta - 1) - 4b\delta)} > -c(\delta - 1) \\
\iff c^2(\delta - 1)^2 - 4bc\delta(\delta - 1) > c^2(\delta - 1)^2 \\
\iff -4bc\delta(\delta - 1) > 0 \\
\iff 4bc\delta(\delta - 1) < 0 \]

where the last line is true because $1 - \delta > 0$. Also note that $\epsilon$ is always a real number, since the term underneath the square root is positive if and only if $b > \frac{c(\delta - 1)}{4\delta}$, which is always true since $b > 0$.\(^\Box\)

Proof of Proposition 4.

Proof. Just by taking derivatives, we have that

$$\frac{\partial \epsilon}{\partial b} = \frac{1 - \delta}{\sqrt{c(\delta - 1)(c(\delta - 1) - 4b\delta)}} > 0 \text{ since } \delta < 1$$

$$\frac{\partial \epsilon}{\partial c} = \frac{-b(1 - \delta)}{c\sqrt{c(\delta - 1)(c(\delta - 1) - 4b\delta)}} < 0 \text{ since } \delta < 1$$

Now we focus on $\frac{\partial \epsilon}{\partial \delta}$. By taking derivatives we have

\(^6\)Proven next.
\[
\frac{\partial \epsilon}{\partial \delta} = \frac{c(\delta - 1) - 2b\delta + \sqrt{c(\delta - 1)(c(\delta - 1) - 4b\delta)}}{2\delta^2 \sqrt{c(\delta - 1)(c(\delta - 1) - 4b\delta)}} < 0
\]
\[\iff c(\delta - 1) - 2b\delta + \sqrt{c(\delta - 1)(c(\delta - 1) - 4b\delta)} < 0\]
\[\iff \sqrt{c(\delta - 1)(c(\delta - 1) - 4b\delta)} < 2b\delta - c(\delta - 1)\]
\[\iff c(\delta - 1)(c(\delta - 1) - 4b\delta) < 4b^2\delta^2 - 4b\delta c(\delta - 1) + c^2(\delta - 1)^2\]
\[\iff -4b\delta c(\delta - 1) < 4b^2\delta^2 - 4b\delta c(\delta - 1)\]
\[\iff 4b^2\delta^2 > 0 \text{ which is always true since } b, \delta > 0.\]

\[\square\]

**Proof of Proposition 5.**

*Proof.* This is easy to see from the fact that all parameters in the model are strictly positive. In particular,

\[
p > 0 \iff c(1 - \delta) + 2b\delta > \sqrt{c(\delta - 1)(c(\delta - 1) - 4b\delta)}
\]
\[\iff c^2(1 - \delta)^2 + 4b^2\delta^2 + 4bc(1 - \delta) > c^2(\delta - 1)^2 + 4bc(1 - \delta)
\]
\[\iff 4b^2\delta^2 > 0 \text{ which is always true since } b, \delta > 0\]

\[\square\]

**Proof of Proposition 8.**

*Proof.* The \(\epsilon\) is calculated exactly the same way as it was in Proposition 2. In particular, we need to solve the following equality for \(\epsilon\).

\[
\frac{b - ce}{(1 - \delta)} = \frac{\delta}{(1 - \delta)}(2 \int_x^{x+\epsilon} c(\epsilon - (y - x)) dy + (b - ce)) - \delta \bar{p}
\]

This yields

\[
\epsilon = \frac{c(\delta - 1) + \sqrt{c(\delta - 1)(c(\delta - 1) - 4\delta(b + \bar{p}\delta))}}{2c\delta}
\]

To ensure that \(\epsilon\) does not loop and overlap around the unit circle, we impose the condition that

\[
(6.3) \quad \epsilon = \min\left\{1, \frac{c(\delta - 1) + \sqrt{c(\delta - 1)(c(\delta - 1) - 4\delta(b + \bar{p}\delta))}}{2c\delta}\right\}
\]

Let \(\bar{\epsilon} = \frac{c(\delta - 1) + \sqrt{c(\delta - 1)^2 + 4\delta(b(1 - \delta) + \bar{p}\delta)}}{2c\delta}\). In order to ensure that \(\epsilon \in \mathbb{R}_+\), we need to both check that the term under the square root is nonnegative and that \(\bar{\epsilon}\) is positive. Thus, \(\epsilon\) is real if and only if

\[
c^2(\delta - 1)^2 - 4c\delta(\delta - 1)(b + \bar{p}\delta) > c^2(\delta - 1)^2
\]
\[\iff 4c\delta(1 - \delta)(b + \bar{p}\delta) > 0
\]
\[\iff \bar{p} > -\frac{b}{\delta}\]

\[\square\]
Also, $\epsilon$ is non-negative if and only if
\[
\begin{align*}
    c^2(\delta - 1)^2 - 4\delta(\delta - 1)(b + \bar{p}\delta) & > 0 \\
    \iff (b + \bar{p}\delta) & > -\frac{c(\delta - 1)^2}{4\delta(1 - \delta)} \\
    \iff \delta\bar{p} & > -\frac{b - c(1 - \delta)}{4\delta} \\
    \iff \bar{p} & > -\frac{b}{\delta} - \frac{c(1 - \delta)}{4\delta^2}
\end{align*}
\]
(6.5)

Since $\frac{c(1 - \delta)}{4\delta^2} > 0$, we have that (6.4) implies (6.5). Thus, we assume in the model that the optimal $\bar{p}$ obeys condition (6.4).

Proof of Proposition 10.

Proof. We proceed simply by taking derivatives.
\[
\frac{\partial \epsilon}{\partial \bar{p}} = \frac{\delta(1 - \delta)}{\sqrt{c(\delta - 1)(c(\delta - 1) - 4\delta(b + \bar{p}\delta))}} > 0 \quad \text{since } \delta > 0
\]

REFERENCES